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Predicting Pore Pressure and Applications to Complex Loading Problems

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SYNOPSIS Methods for predicting and modeling the pore pressure response when soils are subjected to cyclic loading are described. These predictions include both the limiting states of pore pressure and the intermediate stages of development as a function of number of cycles. Applications of pore pressure prediction in effective stress methods are illustrated for a group of problems including sequencing of irregular loading, degradation of stress-strain modulus, volume changes and strength changes.

INTRODUCTION

The response of soils to cyclic loading encompasses a broad range of behavior and includes a group of problems involving stress-deformation, strength and time effects. Most published work on this subject has been directed toward one particular aspect or another, for example, degrading stress-strain behavior or the relationship of cyclic stress level to the onset of liquefaction. A general, integrated behavior model of cyclic loading offers several advantages, not only for predicting behavior but also to establish relationships among the various parts of the overall behavior which are usually considered separately. In this paper an integrated fundamental model is described.

Change in pore pressure is the fundamental response of soils subjected to undrained loading. Shearing resistance, stress-strain behavior and changes in these two properties with time depend upon the state of pore pressure and its dissipation. Effective stress methods are especially useful in complex problems such as those involving partial drainage or repeated loading. The key to these methods is the predictability of pore pressure. In this paper the methods for predicting excess pore pressure during repeated loading are applied to the problem of predicting the response to a sequence of cycled loads of different magnitudes and to the problem of predicting degradation of the stress-strain modulus during cyclic loading. Other applications are discussed.

PREDICTING PORE PRESSURE

To characterize completely the excess pore pressure resulting from cyclic loading requires consideration of all the following:

1. the maximum amount of excess pore pressure potential associated with a particular level of loading

2. consideration of whether this level of loading and excess pore pressure leads to failure or nonfailure equilibrium, given a very large number of cycles

3. the rate of developing excess pore pressure with strain or number of cycles

4. the dissipation of excess pore pressure through drainage

For contractive soils (positive excess pore pressures), which are the major concern in practice, typical data are shown in Fig. 1. The limiting pore pressure states, Δu_{max}, have been the objective of studies by several groups (Martin et al., 1975; Egan and Sangrey, 1978; Seed, 1979) and there is some consensus that these limiting conditions can be predicted on the basis of fundamental soil properties. The predictability of intermediate pore pressure is less certain.

A typical set of test data, Fig. 2, illustrates the range of intermediate pore pressures associated with undrained cyclic loading of a contractive soil, in this case a saturated silty clay. The accumulating excess pore pressures, $\Delta u_{r,i}$, have been normalized by the maximum pore pressure potential for this soil

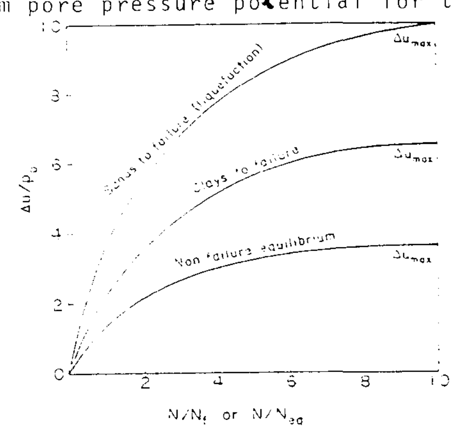

Fig. 1. Alternative forms of pore pressure response to cyclic loading

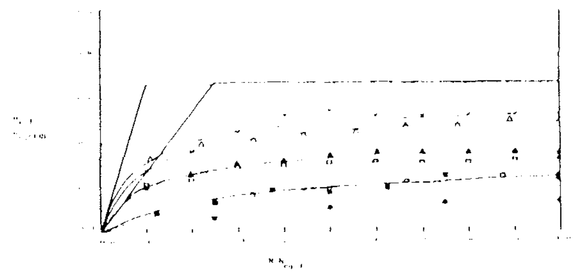

Fig. 2. Variety of pore pressure response resulting from various levels and types of cyclic loading

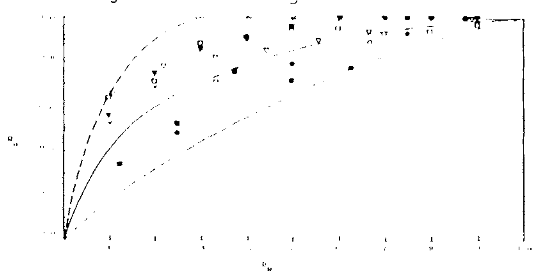

Fig. 3. Normalized pore pressure data for a set of tests representing a variety of test conditions

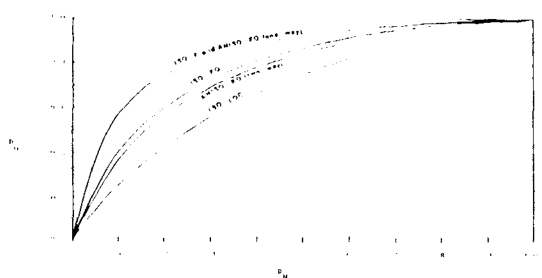

Fig. 4. Pore pressure response curves for individual subsets of the data from Fig. 3.

which is the pore pressure at the critical level of repeated loading, $\Delta u_{r,CLRL}$. The number of cycles for each test specimen has been normalized by the number of cycles required to reach either equilibrium, N_{eq} or failure, N_f. The nine test specimens were consolidated under an anisotropic stress condition and subject to different levels of cycled triaxial compression. Two specimens cycled at the higher stress levels failed (solid lines); four specimens cycled with an increasing compressive stress only reached non-failure equilibrium (open symbols); and three specimens cycled with both a relative compression and tension component also reached non-failure equilibrium conditions (solid symbols).

These data are typical in showing different specific levels of excess pore pressure for different loading conditions but with an overall similarity in the shape of the curve or rate of intermediate pore pressure increase. Mathematical equations fitting empirical data such as those in Fig. 2 have been proposed (Rahman, Seed and Booker, 1977; Sangrey, 1979) and Lascko (1980) has done a comprehensive review to assess the relative advantages of these different proposals. There is no theoretical or physical reason why any empirical equation should be better than another. Rahman, Seed and Booker's equations will have a concave upward hook at the upper end which is similar to some laboratory test data on sands. Hyperbolic or exponential equations have the advantage of simplicity and appear to fit most laboratory and field data. The curves shown in Fig. 2 are all of hyperbolic form.

There are distinct advantages to having empirical data presented in a form minimizing the number of variables which must be considered. For a set of data from the same soil and initial conditions, Lascko (1980) proposes to eliminate the level of cycled stress by normalizing the pore pressure by the predictable end condition, Δu_{max}. The resulting parameter, the pore pressure ratio, is:

$$R_u = \frac{\Delta u_{r,n}}{\Delta u_{max}} = \frac{\Delta u_{r,n}}{\Delta u_{eq-f}} \qquad \ldots . 1$$

where Δu_{max} can be either the equilibrium pore pressure, Δu_{eq}, or the pore pressure at failure, Δu_f. A form of the pore pressure ratio which is particularly useful (see following section) is based on normalization using the excess pore pressure at the critical level of repeated loading. This formulation will be defined as:

$$R_u{}^* = \frac{\Delta u_{r,n}}{\Delta u_{r,CLRL}} \qquad \ldots . 2$$

Another useful normalizing parameter is the number of loading cycles ratio:

$$R_N = N/N_{eq-f} \qquad \ldots . 3$$

Using the normalized parameters, the individual data shown in Fig. 2 can be considered as a single population. These data are presented in Fig. 3 along with a statistical best fit using a hyperbolic equation:

$$R_u = \frac{R_N}{a+bR_N} \qquad \ldots . 4$$

a and b are empirical constants (Sangrey, 1979). Bounds for the data are shown using dashed lines. These relationships are typical of many data sets examined by Lascko and show that for a particular soil a single equation can be used to describe the rate of pore pressure change between know limits. The results in Fig. 3 also illustrate that the reliability of predictions is only ·35 in some cases. This error is primarily a consequence of experimental or testing variables such as rate of testing, testing equipment (triaxial or simple shear), type of cyclic stress (one or two directions), shape of stress pulse and others which influence the rate of pore pressure development. The limits of pore pressure, Δu_{max}, are not affected by these loading and testing variables to the same degree as the intermediate pore pressure.

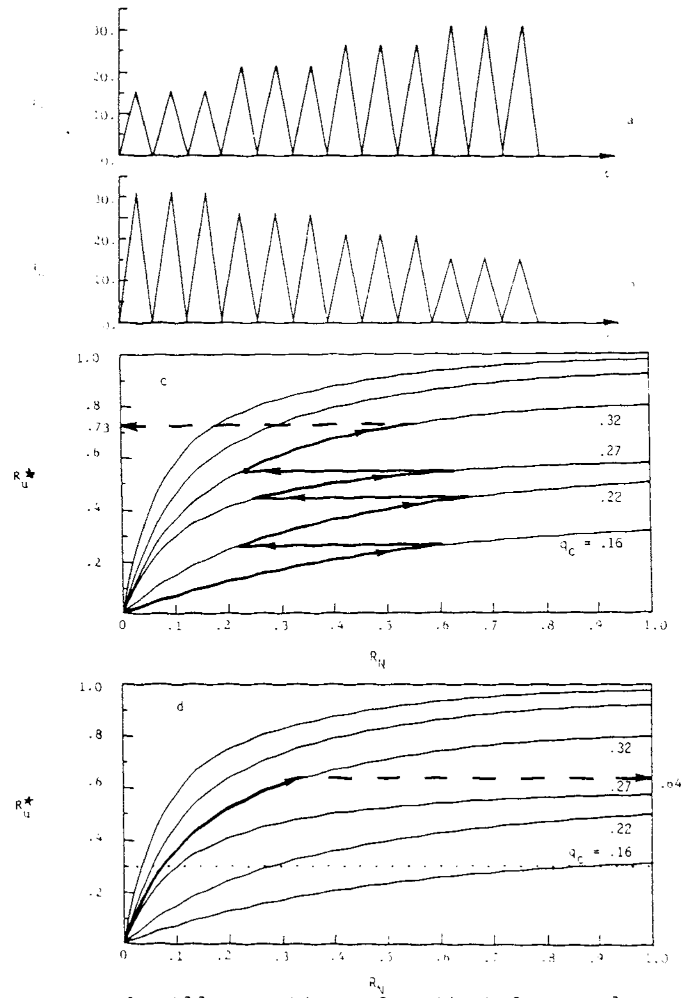

Fig. 5. An illustration of method for evaluating a sequence of loading
a,b. the loading functions
c. response of loading sequence a
d. response of loading sequence b

The accuracy of the prediction presented in Fig. 3 can be improved by separating the data set into subsets. For example the effect of stress history and type of cyclic stress has been used to separate the data illustrated in Fig. 4. In this case each of the hyperbolic curves fits the data with an accuracy of better than ±20% in contrast to the attempt at a universal model with an accuracy of ±35% (Fig. 3).

APPLICATIONS OF PORE PRESSURE PREDICTION

a. Volume change and strength change

The amount of excess pore pressure resulting from cyclic loading of soils has been linked to the potential for volume change. France and Sangrey (1976) illustrated the predictability of volume change on this basis and extended their work in a method for predicting strength changes in soils experiencing any combination of drained and undrained repeated loading (Sangrey and France, 1980). Other effective stress applications are described below.

b. Sequencing

The data presented in the previous figures have been based on cyclic loading with a constant level of cycled stress. In many real problems the level of cycled stress varies within the loading sequence. When this is the case the empirical curves and the limiting pore pressure predictions are not directly applicable.

A method for considering sequencing of cyclic loading of different stress levels can be based on curves of intermediate pore pressure such as those shown in Fig. 2, 3 and 4. The basic hypothesis of the method is that, for a particular soil, the response to an individual cycle of stress depends only on the effective stress state at the time of application of that stress increment and not on the stress history. The effective stress state is directly associated with the amount of accumulated excess pore pressure. If this hypothesis is combined with the fact that there is a maximum potential excess pore pressure for any soil and that this pore pressure occurs at the critical level of repeated loading, $\Delta u_{max} = \Delta u_{r,CLRL}$, then the development of excess pore pressure between 0 and $\Delta u_{r,CLRL}$ can be considered a measure of how close the soil is to failure or how much "damage" has been done to the soil by cyclic loading. The amount of accumulated pore pressure, or damage, done to a soil by a particular combination of cycled stress level and number of cycles, R_N, would then be equivalent to all other combinations of cyclic stress level and R_N which would produce the same amount of pore pressure change. This principle is illustrated in Fig. 5d where the pore pressure value of $R_u^* = 0.3$ could have been achieved by any combination of cycled stress level and R_N intersected by the dotted line. Furthermore, the response of the soil to subsequent loading would be no different regardless of which combination of cycled stress and R_N produced the pore pressure of $R_u^* = 0.3$.

When this approach is applied to a sequence of cycled stress of different magnitude, the amount of accumulated excess pore pressure is the measure of the effects of the sequence. The total excess pore pressure will be

$$R^*_{u,t} = \sum_{i=1}^{n} \Delta R^*_{u,i} \qquad \ldots\ldots 5$$

where each $\Delta R^*_{u,i}$ is calculated for a particular cycled stress level, q_c, and number of applied cycles of that stress. The calculated value of $\Delta R^*_{u,i}$ depends on the initial conditions when the sequence begins and, if hyperbolic models are used (equa. 4):

$$R^*_{u,i} = \frac{R_{N,i}}{a_i + b_i R_{N,i}} \qquad \ldots\ldots 6$$

Application of the principle outlined above is illustrated in Fig. 5. The two complementary loading functions shown in Fig. 5a and 5b each contain the same number and magnitude of stress cycles but in reverse order. The hyperbolic curves shown in Fig. 5c and 5d correspond to the applied cycled stress levels. The predicted intermediate pore pressure for the loading in Fig. 5a is illustrated in Fig. 5c where the first three cycles of q_c = 0.16 are equivalent to R_N = 0.6 and an excess pore pressure of $R_u{*}$ = 0.23. To consider the effect of the next set of three cycles of q_c = 0.22 requires describing the loading up to this point in terms of loading at a stress level of 0.22. This equivalency would be R_N = 0.23 which corresponds to 1.58 cycles of stress at this level. Adding the additional three cycles of q_c = 0.22 would bring the response to $R_u{*}$ = 0.42, R_N = 0.65. An equivalency for the consideration of the next level of cyclic loading, q_c = 0.27, would be $R_u{*}$ = 0.42, R_N = 0.25 and so forth. The predicted pore pressure at the end of this loading sequence would be $R_u{*}$ = 0.73.

The significance of the sequence of cyclic loading is illustrated in the Fig. 5d where the complementary loading sequence is applied, Fig. 5b. In this case the final pore pressure is $R_u{*}$ = 0.64 and all of this excess pore pressure or damage resulted from the first three loading cycles at q_c = 0.32. All subsequent loading cycles added no additonal excess pore pressure. As illustrated in Fig. 5, the order of a sequence of irregular stress cycles can lead to significantly different response.

c. Application to Modulus Degradation

The key soil property for deformation analysis is a stress-strain modulus. When loading is repetitional, deformation analysis is complicated because the modulus is both nonlinear within a cycle but also may degrade from one cycle to another. Idriss, et al. (1978) have used a Ramberg-Osgood function as the basis for a degrading model. Their work is typical of dynamic modulus studies for soil which do not make explicit use of the effective stress state of the soil. However, the methods for predicting pore pressure described above can provide a fundamental basis for modeling degradation.

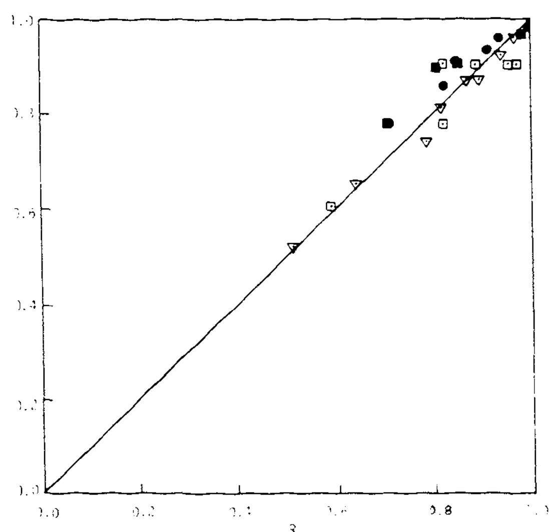

Fig. 6. Relationship between accumulated pore pressure and modulus degradation

Idriss, et al. (1978) use a degradation index:

$$\delta = (E)_N/E_1 = N^{-t} \qquad . . . . 7$$

defined as the ratio of modulus at a particular cycle to initial modulus. They also consider δ to be empirically related to the number of cycles through the degradation parameter, t. In work to establish a fundamental relationship between excess pore pressure, Lascko (1980) introduced a normalized degradation ratio:

$$R_d = \frac{1 - \delta_N}{1 - \delta_{eq-f}} \qquad . . . . 8$$

where the denominator is defined as the degradation index at either equilibrium or failure.

The data in Fig. 6 illustrate that the degradation index, R_d, and excess pore pressures during cyclic loading, R_u, are strongly related. Such a strong relationship would indicate that effective stress methods could be used as a basis for modeling degradation instead of the empirical form of equa. 7. Use of pore pressure offers the advantage of being more fundamental and of being the same reference parameter used to describe other aspects of behavior.

CONCLUSION

The limits and intermediate stages of excess pore pressures developed during undrained cyclic loading can be predicted and modeled. These pore pressure values can be used as the basis for effective stress methods applicable to predictable volume changes, strength changes, and modulus degradation under cyclic loading.

ACKNOWLEDGMENTS AND REFERENCES

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